Resources
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Archimedes Principle

Any object, wholly or partly immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object.

Some objects, when placed in water, float, while others sink, and still others neither float nor sink. This is a function of buoyancy. We call objects that float, positively buoyant. Objects that sink are called negatively buoyant. We refer to object that neither float nor sink as neutrally buoyant.

The idea of buoyancy was summed up by Archimedes, a Greek mathematician, in what is known as Archimedes Principle: *Any object, wholly or partly immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object.*

From this principle, we can see that whether an object floats or sinks, is based on not only its weight, but also the amount of water it displaces. That is why a very heavy ocean liner can float. It displaces a large amount of water.

Archimedes principle works for any fluid, but as divers we are mainly concerned with two different fluids: fresh water, and salt water. We need to think of fresh water and salt water as two different fluids because equal volumes of fresh water and salt water do not weigh the same. For example, a cubic foot of fresh water weighs approximately 62.4 lbs, while a cubic foot of salt water weighs approximately 64 lbs. The extra weight is because of the dissolved minerals in salt water.

Let's take a moment and look at an object in water and Archimedes Principle. If you place a 1 cubic foot object that weighs 63 lbs into fresh water, the object is displacing 62.4 lbs of water, but weighs 63 lbs. This object will be negatively buoyant - it will sink. It is however being buoyed up with a force of 62.4 lbs, so if we weighed it in the water it would only weigh .6 lbs.

If we put the same object into salt water, it would still weigh 63 lbs, but would be buoyed up by a force of 64 lbs, and it would float. It would be positively buoyant in salt water. To make the object neutrally buoyant in salt water, we would have to add 1 lb of weight to the object without changing its size (without changing is displacement). Then it would weigh 64 lbs, and be buoyed up with a force of 64 lbs, thus being neutrally buoyant.

Let's expand on this and look at an example of putting these ideas to work in a real situation. Suppose you know that a boat had lost an anchor weighing 100 lbs. measuring a comparable anchor, we find out that the anchor displaces 1/2 cubic feet of water. We will also assume that the anchor was lost in a fresh water lake. You do a dive and find the anchor and want to bring it to the surface but the only resource you have available are some 1 gallon milk jugs. How many would you need to tie on to the anchor to float it to the surface?

At this point we need to do a little simple math. We know that a cubic foot of fresh water weighs 62.4 lbs, so the anchor displacing 1/2 a cubic foot of water would be buoyed up with a force of 31.2 lbs. Let's round this to 31 lbs for simplicity. This means our anchor that weighs 100 lbs on land will weigh 100-31 or 69 lbs in the water. We now know we need enough 1 gallon milk jugs to generate 69 lbs of lift.

Perhaps you remember the old expression "A pint a pound the world around." This refers to the fact that a pint of water weighs about a pound. Since there are 8 pints in a gallon, we know a gallon of water must weigh about 8 lbs. Since we know a cubic foot of water weighs 62.4 lbs, this means there are about 8 gallons of water in a cubic foot. Let's put it together and solve our anchor problem.

If we need 69 pounds of lift, we divide 69 by 8 lbs per gallon to learn we need 8.625 gallons of water displacement to make the anchor neutrally buoyant. This means, we could fill 9-one gallon milk jugs with air to lift our anchor.

Let's try another. A 3 cubic foot object weighing 400 lbs is dropped into the ocean. How big of an air lift bag (in cubic feet) would you need to lift the object?

First we determine that a 3 cubic foot object in salt water would have 3x64 lbs of lift, or 192 lbs of buoyant force. If we subtract 192 from 400 we get 208 lbs. This means we need to generate 208 lbs of lift to make our object neutrally buoyant. We then divide 208 (the objects in water weight) by 64 (the weight of a cubic foot of sea water) to get 3.25 cubic feet of displacement is needed to make the object neutrally buoyant. Thus, we would need at least a 3.25 cubic foot air lift bag.

3 c.f object 400lbs, salt water (64lbs c.f)

64c.f x 3c.f=192c.f displacement, object weights 208lbs in the water

208lbs/64lbs per .c.f= 3.25 c.f to make buoyant

Pressure and Gases

It has long been known that air has weight. The weight of the atmosphere exerts a pressure of 14.7 pounds per square inch of force at sea level. This is to say that a 1 inch column of air as tall as the atmosphere, would weigh 14.7 pounds. We commonly call this 1 Atmosphere of pressure, or 1 ATM.

Water weighs considerably more than air does, so it can exert much more pressure. It only takes a 1 inch column of sea water 33 feet tall to weigh 14.7 pounds. This means that at a depth of 33 feet deep in the ocean, there is a total pressure of 29.4 pounds per square inch (psi). This would be 2 ATMs of pressure. One ATM from the water, + one ATM from the atmosphere. We call this the ambient pressure or absolute pressure.

Every additional 33 feet of sea water, will add another 14.7 pounds of pressure, or another ATM. At 66 feet we have two ATMs of water pressure plus our 1 ATM of air pressure for an absolute pressure of 3 ATM. At 99 feet our pressure is 4 ATMs etc.

Absolute pressure differs from gauge pressure. Gauge pressure would be the pressure that would show up on a gauge at this depth. A pressure gauge would start at 0 at the surface and show 14.7 psi at our depth of 33 feet. Gauge pressure would ignore the 14.7 psi of atmospheric pressure.

And now for a little math!

With a little math, we can determine the ambient pressure at any given depth. Knowing that 33 feet of sea water exerts a pressure of 14.7 lbs, it is simply a matter of dividing 14.7 by 33 to determine that sea water exerts a pressure of .445 lbs per foot of depth. Fresh water being slightly lighter, requires a depth of 34 feet to equal 1 ATM, so fresh water exerts a pressure of 14.7 divided by 34, or .432 lbs per foot of depth.

Let's then find out the absolute pressure for a depth of 50 feet of sea water. To solve this, we multiply the depth, 50, times the pressure per foot, .445. This gives us an answer of 22.25. This however would be the gauge pressure at a depth of 50 feet. To find the absolute pressure, we must add in the 14.7 psi of atmospheric pressure. 22.25 + 14.7 = 36.95 Thus the pressure for a depth of 50 feet in sea water is 36.95 pounds per square inch absolute. It is common to refer to this as "PSIA", pounds per square inch absolute.

Try to figure out the gauge pressure for a depth of 75 feet in fresh water. To solve this we multiply .432 by 75 to get 32.4 psi. Since we wanted gauge pressure in this example, we do not add in the atmospheric pressure and show the pressure using psi, not psia.

Intro to Boyle's Law
Pressure Volume Relationships

We are used to living at 1 ATM of pressure, so we rarely even take notice of it. We normally don't feel the pressure on us because the human body is primarily made up of liquid, and liquids are basically non compressible. At times, however, we do notice changes of pressure, primarily in our ears. You may have noticed your ears "popping" when flying, driving in the mountains, or even going up and down in elevators. This is because our ears have an air space in them, and air, like all other gases, is compressible.

A gas will compress proportionately to the amount of pressure exerted on it. For example, if you have a 1 cubic foot balloon and double the pressure on it, it will be compressed to 1/2 cubic foot. Increase the pressure by 4, and the volume will drop to 1/4 the size etc. This theory was discovered by Sir Robert Boyle, a 17th century scientist. The theory known as Boyle's Law states: *If the temperature remains constant, the volume of a given mass of gas is inversely proportional to the absolute pressure.*

Let's follow an example...

Suppose you had a balloon measuring one cubic foot at the surface of the water. This balloon is under 1 ATM (14.7 psi) of pressure. If we push the balloon underwater, and take it to a depth of 33 feet, it is now under 2 ATM of pressure (29.4 lbs) - 1 ATM of pressure from the air, 1 ATM of pressure from the water. Boyle's Law then tells us that since we have twice the absolute pressure, the volume of the balloon will be decreased to one half. It follows then, that taking the balloon to 66 feet, the pressure would compress the balloon to one third its original size, 99 feet would make it 1/4 etc.

If we bring the balloon in the previous example back up to the surface, it would increase in size due to the lessening pressure until it reached the surface and returned to its one cubic foot size. This is because the air in the balloon is compressed from the pressure when submerged, but returns to its normal size and pressure when it returns to the surface.

We will achieve the same result with an open container, such as an inverted bottle, as we do with a balloon. By inverting a bottle at the surface and descending with it, the pressure from the surrounding water will compress the air and the bottle will start to fill with water. Even with no air escaping, the container will be half full of water at a depth of 33 feet due to the pressure compressing the air to half its original volume.

Along with the volume of air in the balloon or container, the surrounding pressure will affect the density of the air as well. Density, simply stated, is how close the air molecules are packed together. The air in the balloon or container at the surface is at its standard density, but when we descend to the 33-foot level where its volume is reduced to one half, the density has doubled. At 66 feet, the density has tripled. This is because the pressure has pushed the air molecules closer together.

Let's continue with this line of thinking and try an additional experiment. If we take our balloon and our open container down to 99 feet, we know that the density of air is four times what is was on the surface and the volume of air has been reduced to 1/4. Now at this depth, suppose we used a scuba tank and added air to the balloon until it returned to its original size. We will also blow air into the inverted container until it is completely full of air.

We know the air at this depth is 4 times denser than at the surface. This means when we ascend with our balloon and container, the lessening pressure will make the air expand. This will have two different effects. The balloon will increase in size. It will attempt to grow to a size of 4 cubic feet by the time it hits the surface. If this is beyond the capability of the balloon, it will pop. The inverted container, however, will simply "bleed off" the expanding air that will harmlessly float away as bubbles.

The main purpose of the proceeding material was to give you the theory behind the most important rule in scuba diving... "Never hold your breath!" Your lungs can act very much like a pair of balloons in your chest. As a breath hold diver (skin diver), if you fill your lungs with air at the surface, hold your breath, and dive to a depth of 33 feet, the surrounding pressure will compress your lungs to half of their original size. Upon ascending, they will return to normal size. If however, you descend to 33 feet and breath compressed air from a scuba tank, an ascent to the surface could cause you lungs to over expand and you could seriously injure yourself.

This is easy to avoid, however, by simply not holding your breath which will let your lungs act like the open container in the preceding example, and you will simply "bleed off" the expanding air and maintain a normal lung capacity.

Advanced Boyle's Law
Pressure Volume Relationships

We can take the knowledge we have gained thus far and figure out the volume of objects at any depth, not just at 33 feet, 66 feet etc. Boyle's Law tells us that there is an inversely proportional relationship between the pressure and the volume of gases. This means that the pressure (P) exerted on a gas times the volume (V) of the gas will always equal a constant (K). P x V = K.

If we take this one step further, we know that since this holds true for our balloon at any depth, the pressure times the volume at one depth must equal the pressure times the volume at any other depth. Or:
P1 x V1 = P2 x V2
where P1 is the pressure at the first depth and V1 is the volume at the first depth and P2 is the pressure at the second depth and V2 is the volume at the second depth.

Let's plug some numbers into this equation to see how it works. To make our first example easy, let's take an example we have already done. A balloon is 1 cubic foot at the surface, how big would it be it we took it down to 66 feet. We know the answer should be 1/3 cubic foot, but let's work through the formula.

The pressure at the surface is 14.7 psi and the volume is 1 cubic foot so the first half of our equation looks like:

14.7 x 1 = P2 x V2

Next we need to determine the absolute pressure at our second depth. (If you don't know about absolute pressure, check out Into to Gasses.) To calculate the absolute pressure at 66 feet we multiply 66 times the pressure per foot in salt water, .445 and then add in 14.7 psi to give us the absolute pressure at this depth.

P2 = (66 x .445) + 14.7

P2 = 29.37 + 14.7 or

P2 = 44.07

Plugging the numbers into our formula then, we get

14.7 = 44.07 x V2

To solve for V2, we divide both sides of the equation by 44.07 that gives us the following:

14.7 / 44.07 = V2

Solving for this we then see that

V2 = 14.7 / 44.07 or 0.333 or 1/3.

Let's try another problem. If a balloon is 1 cubic foot in 20 feet of fresh water, how big would it be at a depth of 50 feet?

We use our formula P1 x V1 = P2 x V2 and start putting in the numbers. We know our V1 is 1. To determine P1 we multiply 20 times the pressure per foot in fresh water .432, then add 14.7. P2 would be 50 times .432 + 14.7 so our equation looks like this:

[(20 x .432) + 14.7] x 1 = [(50 x .432) + 14.7] x V2

Using a calculator we start doing the math:

[8.64 + 14.7] = [21.6 + 14.7] x V2

we add up the sides to give us:

23.34 = 36.3 x V2

Then we divide both sides by 36.37 to get:

23.34 / 36.3 = V2

or V2 = 0.6429752066116

rounding this number, we see that a 1 cubic foot balloon at 20 feet would be compressed to about .64 of a cubic foot at a depth of 50 feet.

As you can see, it is simply a matter of punching the numbers into the equation derived from Boyle's Law to calculate volumes and pressures of compressible objects at any depth.

Surface Air Consumption Rate

The compressibility of gasses is also an important consideration for divers due to its affect on how long a diver can stay underwater. Scuba regulators are designed to deliver air to a diver at the same pressure as the surrounding water pressure, at ambient pressure. That means that when a diver fills his lungs at a depth of 33 feet, he is taking in the equivalent amount of air as two breaths at the surface. Obviously then, a tank will only last half as long at 33 feet as it would at the surface. And tank that would last 1 hour at the surface would only last 1/3 as long, or 20 minutes, at a depth of 66 feet, etc.

It can be beneficial to be able to estimate how long a scuba tank might last at a given depth when dive planning. To determine this, it is first necessary to determine a divers Surface Air Consumption (SAC) rate. For example, if you are diving at 33 feet, and use 500 lbs of air in 10 minutes, it is easy to determine that you are using 50 lbs per minute. This is only true for this depth however. How much air would you use at 66 feet, or 99 feet?

The first thing we must do is calculate SAC rate. In the preceding example, a diver using 50 pounds per minute at a depth, would use 25 pounds per minute at the surface. His surface air consumption rate is 25 pounds per minute. With our SAC number of 25, it is easy to calculate our consumption rate for depths of 33, 66, 99 feet etc. We know we are under 3 times the pressure at 66 feet, so we would use 3 times as much air, or 75 pounds per minute at this depth.

The process becomes slightly more complex if depth consumption rate (DCR) is determined at a depth that is not in even atmospheres. (Not at 33, 66, 99 feet etc.) For this situation we use a formula that is simply an adaptation of Boyle's Law to determine our SAC rate:

SAC Rate = (DCR x 33) / (Depth + 33)

Let's look at an example. Suppose you did a 50 foot dive for 25 minutes and used 1700 pounds of air. This would mean our DCR is 1700/25 or 68 pounds per minute. Using this in our formula we get:

SAC Rate = (68 x 33) / (50+33)

or: SAC Rate = 2244/88 or 25.5 pounds per minute.

We can then turn the equation around to determine our DCR for any depth.

DCR = SAC Rate x (Depth + 33)/33

Let's assume our SAC Rate is 25 and we want to know how fast will we use 2000 pounds of air at a depth of 75 feet.

Dropping our numbers into the equation we get: DCR = 25 x (75 + 33)/33 or DCR = 25 x 108/33 or DCR = 81.81

This means at a depth of 75 feet, we will use 81.81 pounds of air per minute. Dividing this into the 2000 pounds, we see this amount of air would last 24.4 minutes.

It is important to note that SAC Rate takes into account the assumption that you are exerting the same amount of energy at any given depth, and you are using the same size tank as you used when calculating your DCR.

For example, under strenuous diving conditions, you can consume air 4 times faster than when sitting still taking pictures. Also it is obvious that a 50 cubic foot tank would not last as long as an 80 cubic foot tank, even if they were both filled to the same pressure

Charles' Law
Gas Volumes and Heat

There is another way to make a balloon smaller other than pushing it underwater. You can put it in the freezer. When you heat up a gas, the molecules that the gas is made up of move faster. In our balloon example, this increase in molecular motion causes the molecules to hit the sides of the balloon more often, and with more force, making the balloon expand. Cooling the gas would have the inverse effect, making the balloon smaller.

While not too important when dealing with balloons, this concept has other applications. For example a full scuba tank, if left in the sun, will heat up. This causes the molecules in the air in the tank to move faster. Unlike the balloon which would expand, the tank is a rigid container that will not expand. This increase in motion then raises the pressure inside the tank. In fact, a full scuba tank will gain about 5-6 psi for every degree of temperature increase.

This is one reason that full tanks should not be left in a hot trunk of a car. A tank filled to 3000 psi could easily reach 3500 psi if the temperature of it increased substantially. There have also been several cases where full scuba tanks, involved in boat fires, have exploded. This is due to the weakening the metal and the increased pressure from the heat.

If go back to Boyle's Law and read a little more carefully we will see that Boyle's Law states: If the temperature remains constant, the volume of a given mass of gas is inversely proportional to the absolute pressure. Boyle did not concern himself much with changing temperature. This was, however, the main goal of a French scientist Jacques Charles.

Charles showed that raising the temperature of a gas would tend to increase the volume of the gas, if its pressure remained constant. A few other laws, like Amonton's Law, "The pressure of a fixed amount of a gas maintained at a constant volume is directly proportional to the gas pressure." or P µ T, P/T = constant or P1/P2 = T1/T2 all came together to help make what has come to be known as the General Gas Law:

$$\frac{P1 \times V1}{T1} = \frac{P2 \times V2}{T2}$$

T1 is the temperature at the first location, T2 is the temperature at the second. Remember when we were dealing with the pressure, we used the absolute pressure for our calculations. We must start at the zero mark. As stated in the Charles' law we must also use absolute temperature in the calculations as well. Absolute zero(** Raising the temperature of a gas causes the molecules in the gas to move faster. Lowering the temperature causes the molecules to slow down. Absolute zero is the theoretical temperature where all molecular motion stops.) is 460 degrees below Fahrenheit zero. This means we must add 460 to our temperatures before applying them to the formula. This is known as the Rankine scale. For example, 40 degrees Fahrenheit would be 460 + 40, or 500 degrees Rankine.

Using the formula, we can explore the second half of Charles' law. This part of the law states that if the volume was kept constant, raising the temperature would increase the pressure of the gas. A good example of this can be found by an example with a scuba tank. Let us look at a scuba tank that shows it is filled to 3000 psi. We will assume the tank reads this pressure while sitting in an air-conditioned room which is at 70 degrees Fahrenheit. How much pressure would there be in the tank if it were left in a trunk of a car where the temperature climbed to 140 F?

Let's put our numbers into the formula. The first thing we can do is take the V's out of the formula. Since the volume of the scuba tank will remain the same, we can cancel the V's and change our formula to: P1 / T1 = P2 / T2

Our starting pressure, P1, at first appears to be 3000 psi. We must remember to use absolute numbers though, so to obtain absolute pressure, we add in atmospheric pressure of 14.7. P1 = 3000 + 14.7. T1 would be our starting temperature in degrees Rankine. T1 = 460 + 70. And T2 would be 460 + 140. Let's now look at our formula with its numbers in place:

$$\frac{3000 + 14.7}{460 + 70} = \frac{P2}{460 + 140}$$

adding up our numbers we get:

3014.7/530 = P2/600

We then multiply both sides of the equation by 600 to get the P2 on one side by itself.

(600 x 3014.7) / 530 = P2 or P2 = 3412.8

It is important to note that 3412.8 is the absolute pressure at the second location. If we were asked "What is the gauge pressure at the second location?" we would subtract 14.7 from 3412.8 for an answer of 3398.1. This is a substantial increase in pressure. It turns out that a full 80 cubic foot scuba tank will have a pressure change of approximately 5-6 psi for every degree of temperature change.

Since Charles' law also deals with changing the volume of a gas with a change in temperature, we can use the General Gas Law formula to determine the answer to the following question. If a balloon is inflated to one cubic foot at the surface, with air that is 85 degrees, how large would the balloon be if taken into 50 degree sea water to a depth of 40 feet?

Solving for our Ps, Vs, and Ts, we get:

P1 = 14.7
V1 = 1
T1 = 85 + 460
P2 = [40 x .445] + 14.7
T2 = 460 + 50

Using these numbers, we can solve for V2.

(14.7 x 1) / 545 = ({[40 x .445] + 14.7} x V2) / 510

solving further we get:

14.7 / 545 = (32.5 x V2) / 510

Since we want to get the V2 by itself on one side of the equation, we will multiply both sides by 510 over 32.5. This will leave the V2 alone on the second side:

(510 x 14.7) / (32.5 x 545) = V2

using a calculator we get:

7497 / 17712.5 = V2

or: V2 = 0.4232604093155

Thus the volume of our balloon at its second location would be about .42 cubic feet.

Dalton's Law
Partial Pressures

Throughout our discussions thus far, we have been talking about the effect of pressure on air. We have discussed air in balloons, air in a tank, and air in a divers lungs. It is important to point out that air is a mixture of many different gasses, but mainly nitrogen and oxygen.

The air mixture is approximately 78% nitrogen, and 21% oxygen with the remaining 1% being a mix of argon, carbon dioxide, neon, helium and other rare gases. While some recreational diving is done on special mixtures like nitrox, most is done breathing plain air. While the fact that air is a mixture of gases is important when we deal with the physiology of diving, we will spend a few moments now to understand the physics of gas mixtures.

It was the English scientist John Dalton that studied the properties of gas mixtures as they relate to pressure and developed Dalton's Law. Dalton's Law states: The total pressure of a gas mixture equals the sum of the partial pressures that make up the mixture.

To study this law as it relates to scuba divers, let's see how this law affects air at different pressures. In order to make our numbers a little more manageable, we will assume that air is a mixture of just two gases, nitrogen and oxygen. We will also assume that the mixture is comprised of 80% nitrogen and 20% oxygen.

If we then look at this mixture as it relates to Dalton's Law, we know that 80% of the pressure of the gas is due to the nitrogen in the mixture and 20% of the pressure is due to the oxygen in the mixture. We refer to these as partial pressures. This means at the surface, the pressure exerted on us by the nitrogen in the air mixture is 80% of 14.7, or 11.76 pounds per square inch. The pressure from the oxygen is 2.94 psi. Together, these account for the 14.7 psi of pressure at the surface.

If we look at pressures at varying depths, we get the following chart:

Partial Pressures of Compressed Air
(assuming air is 80% nitrogen, 20% oxygen)

Depth	Atmosphere	Absolute Pressure	Oxygen Pressure	Nitrogen Pressure
0	1	14.7	2.94	11.76
33	2	29.4	5.88	23.52
66	3	44.1	8.82	35.28
99	4	58.8	11.76	47.04
132	5	73.5	14.70	58.80
165	6	88.2	17.64	70.56
198	7	102.9	20.58	82.32
231	8	117.6	23.52	94.08
264	9	132.3	26.46	105.84
297	10	147.0	29.40	117.60

We see then that as we increase the pressure on us by descending, we are dealing with increased pressure of both nitrogen and oxygen in a 80 - 20 ratio.

It is an easy task to determine the partial pressure of any gas at any depth by using the formulas we have learned thus far. Let's try to determine the partial pressure of oxygen at a depth of 50 feet in sea water assuming oxygen is 20% of the gas mixture.

The first thing we must do is determine the ambient pressure for this depth (If unfamiliar with ambient pressure, refer to Intro to Gas Laws). We know that salt water exerts .445 pounds of pressure per foot, so the water pressure for this depth would be .445 x 50, or 22.25 psi. To this we add the atmospheric pressure of 14.7 for an ambient pressure of 37.95. If we take 20% of this, we have our answer. 37.95 x .20 = 7.39 Thus, the partial pressure of oxygen at a depth of 50 feet in sea water would be 7.39psi.

Determine the partial pressure of nitrogen at a depth of 40 feet in fresh water, while breathing a gas mixture that is 79% nitrogen.

Fresh water exerts .432 psi per foot of depth so we multiply .432 by 40 to get 17.28. To this we add our atmosphere of air pressure, 14.7 to get an ambient pressure of 31.98. If you take 79% of this number you get the answer: 31.98 x .79 = 25.2642